Making HOT lanes benefit every traveler --- design, pricing, and analysis

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A research report from the Pacific Southwest Region University Transportation Center

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The Pacific Southwest Region UTC conducts an integrated, multidisciplinary program of research, education and technology transfer aimed at improving the mobility of people and goods throughout the region. Our program is organized around four themes: 1) technology to address transportation problems and improve mobility; 2) improving mobility for vulnerable populations; 3) Improving resilience and protecting the environment; and 4) managing mobility in high growth areas.

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Disclosure

Principal Investigator, Co-Principal Investigators, others, conducted this research titled, "Making HOT lanes benefit every traveler — design, pricing, and analysis" at the Department of Civil and Environmental Engineering, University of California Davis, and at the Daniel J. Epstein Department of Industrial & System Engineering, Viterbi School of Engineering, University of Southern California. The research took place from 08/16/2020 to 12/31/2021 and was funded by a grant from the United States, Department of Transportation, in the amount of $214,527. The research was conducted as part of the Pacific Southwest Region University Transportation Center research program.

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Abstract

High-occupancy toll (HOT) lanes, as high-occupancy vehicle (HOV) lanes that permit solo-drivers to use them with a charge, play an important role in reducing congestion, vehicle hours traveled (VHT), and vehicle miles traveled (VMT) in the morning commute. In this project, we formulate the morning commute problems with mixed traffic flows on HOT lanes as Differential Complementary Systems (DCS) under two scenarios: (1) two-equilibria model: with total number of solo-drivers and carpoolers fixed, carpoolers reach one equilibrium while solo-drivers reach another (at an equilibrium state, the travel costs of individual commuters are identical); (2) one-equilibrium model: with total number of travelers known, carpoolers and solo-drivers reach one equilibrium. Unlike previous research, our models can jointly determine the departure time choice, lane choice, and/or mode choice. The proposed continuous-time models are converted into discrete-time subproblems. Analytical models are then developed to quantify the time-variant inconvenience costs under equilibrium conditions and set the dynamic toll charges on HOT lanes to avoid queueing. Based on the data from the Caltrans Performance Measurement System (PeMS), the DCS models and analytical models are validated in case studies. Results show that our proposed models can benefit both carpoolers and solo-drivers on HOT lanes: (1) our dynamic tolling approach can avoid congestion on HOT lanes; (2) our model outperforms a modeling approach without tolling; (3) if the HOT lane is underutilized by carpoolers, more solo-drivers would pay to use them; (4) commuters prefer to use carpool when there are more HOT lanes; (5) the overall travel cost, delay, VHT, and VMT per commuter are decreased when more general purpose (GP) lanes are converted into HOT lanes.

Executive Summary

This research investigates the potential benefits of high-occupancy toll (HOT) lanes in the transportation system. It proposes a new design that can (1) promote carpool use while not wasting lane capacity if the lane is underutilized by carpoolers, and (2) benefit both carpoolers and solo-drivers. Differential Complementary Systems (DCS) models are first established to describe the traffic flow evolution and commuters' choices that lead to traffic equilibrium under which the generalized travel cost of commuters of each or all modes are identical. Time-variant inconvenience costs of carpoolers under equilibrium conditions are quantified analytically. Dynamic toll charges for solo-drivers to travel on HOT lanes are proposed to reduce congestion on HOT lanes and demand pressure on general purpose (GP) lanes, with the objective of attracting more solo-drivers to pay to use HOT lanes without producing congestion on the HOT lanes.

Mathematical formulation

We consider the scenario that commuters including carpoolers and solo-drivers want to travel through the freeway system in the morning to reach their work places. These commuters are cost minimizers who choose the best time to depart, best lane to travel on, and/or best travel mode to use in order to minimize their overall travel costs. Two mathematical models are formulated as DCSs:

- Two-Equilibria Model: The total number of carpoolers and the total number of solo-drivers are known, carpoolers only drive on the HOT lane, while solo-drivers can choose to drive on the HOT lane or the GP lane. There are two equilibria in the system: carpoolers on the HOT lane reach one equilibrium; solo-drivers on the HOT lane and solo-drivers on the GP lane reach another equilibrium.

- One-Equilibrium Model: Total number of travelers are known. They can choose to be carpoolers on the HOT lane, solo-drivers on the HOT lane, or solo-drivers on the GP lane. All these three types of travelers reach one equilibrium.

Analysis of time-variant inconvenience costs and dynamic toll charges

We derive the carpoolers' time-variant inconvenience costs analytically under equilibrium conditions. In order to guarantee the quality of service for the HOT lane, we further propose an approach to set the dynamic toll charges to keep the HOT lane operating at capacity and congestion-free:

- For the Two-Equilibria Model: An analytical model is proposed to quantify the inconvenience costs of carpoolers on the HOT lane. In order to spare some space for solo-drivers to pay to use the HOT lane, reserved capacity of solo-drivers on the HOT lane is considered as a model parameter.
- For the One-Equilibrium Model: Dynamic toll charges of solo-drivers on the HOT lanes are considered to reduce every commuter's travel cost and to make them equal, together with the time-variant inconvenience costs of carpoolers.

Numerical experiments and findings

The proposed DCS models and analytical models are solved and calibrated based on data from the Caltrans Performance Measurement System (PeMS). The main results are as follows:

(1) Under the two-equilibria scenario:

- Congestion on HOT lane(s) can be eliminated by setting the toll charge using sensitivity analysis, together with the time-variant inconvenience costs derived under the equilibrium condition.
- The proposed model outperforms a modeling approach without tolling, in terms of system performance such as vehicle hours traveled (VHT), the total delay.
- Under our approach, solo-drivers are more willing to pay and drive on HOT lane(s) when the HOT lane(s) are underutilized by carpoolers.

(2) Under the one-equilibrium scenario:

- By considering time-variant inconvenience costs and dynamic tolling charges jointly, solo-drivers are willing to pay to use the HOT lane without negatively impacting carpoolers. More importantly, HOT lanes remain congestion-free.
- Converting a GP lane to a HOT lane not only attracts more solo-drivers to use it, but also increases the ratio of carpoolers. In total, the number of travelers on the HOT lanes increases.
- Charging the solo-drivers for traveling on the HOT lane does not increase the total commuters' cost. Instead, the average VMT and delay drop significantly, benefiting every traveler in the system.

1 Introduction

Carpooling has been promoted as a way to reduce congestion and vehicle miles traveled (VMT). To facilitate the use of carpool, special lanes, known as high occupancy vehicle (HOV) lanes, have been allocated for the exclusive use of carpool during peak and sometimes nonpeak hours. There are more than 126 HOV facilities on freeways in 27 metropolitan areas in the United States, which includes over 1,000 corridor miles (FHWA, 2017b).

Carpool lanes, however, often face the dilemma of either being underutilized or overly used. In the former, precious road capacity is wasted, and in the later, the carpool lanes no longer provide a travel time advantage over regular lanes. The concept of high-occupancy toll (HOT) lanes was introduced to address the underutilization problem. HOT lanes are HOV lanes that permit solo-drivers to use them with a charge. Such facilities can promote carpool use while not wasting lane capacity if the lane is underutilized by carpoolers, and provide a revenue stream to the HOT lane operators. There are currently over 100 miles operating HOT lane projects in the United States (FHWA, 2017a), 163 corridor-miles HOT/Express lanes under construction (Peterson and MacCleery, 2013) and many states have projects in the planning stages. California is leading the nation with its largest HOT lane system, including I-15 FasTrak in San Diego and I-680 in Alameda County. For example, Caltrans plans to invest up to 779 million dollars to add toll lanes to the 5 Freeway between Red Hill Avenue and the Los Angeles County line (Reyes-velarde, 2019). Recently, an article published in the *Los Angeles Times* [1] titled “A toll lane future is inevitable in California as traffic congestion worsens” points out that, in Orange County, toll lanes on the 91 Freeway save commuters using those lanes up to 30 minutes of travel time per day, according to the Orange County Transportation Authority. Another research study in Washington State shows that carpooling ridership increased by 17% in a new HOT lane corridor and by 23% in the region after its introduction (Burris et al., 2014).

Despite the urgent demand to build and maintain the HOT/HOV lane systems, there are only a few studies that investigate their long term effects on transportation network performance and travelers' behaviour. Wang et al. (2016) studies the influence of HOV lanes on vehicles' optimal route choice and traffic congestion as a pickup and delivery problem with time windows. Di et al. (2017) formulates a user equilibrium to include the presence of HOT lanes, which is solved in Di et al. (2018). The formulation is a continuous bi-level mathematical program with equilibrium constraints, with the lower level problem being the ridesharing user equilibrium. Wang et al. (2019) investigates the optimal capacity allocation for a HOV lane in the morning commute, but does not consider

[1] https://www.latimes.com/

tolling in the HOV lane. Zhong et al. (2020) studies traveler's route choice between a HOT/HOV lane and a regular lane during morning commute. Wang et al. (2020) proposes a feedback control approach to determine the dynamic pricing of HOT lanes.

Morning commute problems with continuous-time bottleneck models are another framework to study HOT/HOV lane problems. Such an approach has been intensively used to study a variety of transportation problems (Pang and Stewart, 2008; Friesz, 2010; Ban et al., 2012a; Ma et al., 2018). They introduce flow propagation in the Dynamic User Equilibrium (DUE) formulation and solves the time-delayed dynamics in the newly developed mathematical model called Differential Complementary Systems (DCS). It integrates ordinary differential equations (ODE) that describe the dynamics of travel time and complementarity conditions that define user equilibrium. For the former, DCS models of single-bottleneck (Ban et al., 2012a) and its extension considering queue spillbacks (Ma et al., 2018) has been well established with convergent solutions, where the "spillback" refers to the queue overgrowing the storage capacity of a link and starts to block the exit flow of upstream links. For the latter, continuous-time DUEs that consider the users' departure-time choices which determine the inflow rates of the travelers.

Previous studies using the DUE morning commute model assume homogeneous commuters (carpoolers or solo-drivers) on a HOT lane at each time. The scenario of mixed traffic flows of both carpoolers and solo-drivers on HOT lane(s) has not been studied to date. Besides, to the best of our knowledge, there is no research to jointly decide commuters' departure time choice, lane choice, and mode choice. Furthermore, how to fully utilize HOT lane capacity without congestion by utilizing dynamic tolling has not been studied under the user equilibrium context.

In this project, we formulate a morning commute problem with different types of travelers (carpoolers and solo-drivers) and lanes (HOT lanes and General Purpose (GP) lanes) under the framework of DCS and complete the model with commuters' departure time choice, lane choice, and/or mode choice under equilibrium conditions. The proposed model is approximated in discretization schemes with time-varying delays. To benefit every commuter with HOT lane(s), we (1) establish the time-variant inconvenience costs analytically for carpoolers on HOT lane(s) under equilibrium conditions, (2) conduct sensitivity analysis to investigate congestion-free toll charges that allow solo-drivers on HOT lane(s) for a two-equilibria model, (3) propose an approach for setting dynamic toll charges to reduce delays and travel costs for every commuter under an one-equilibrium condition, together with time-variant inconvenience costs derived under the equilibrium condition.

The rest of the report is organized as follows. In the next section we formulate the morning commute problems with HOT lanes as DCSs. In Section 3, analytical models are developed for time-variant inconvenience costs and dynamic toll charges on the HOT lanes. In Section 4, the proposed DCS models and analytical models are evaluated in case studies based on data from the Caltrans Performance Measurement System (PeMS). In Section 5, we conclude the report and point out some future research directions.

2 Mathematical Formulation

In this section, we formulate the continuous-time morning commute problem with HOT lanes using Differential Complementary Systems (DCS). Discretizations of the proposed models are also generated for numerical analysis.

2.1 Notations

Notations used in our models are listed below for clarification.

Model Parameters	
D	Total demand of all travelers
D^s	Demand of solo-drivers
D^l	Demand of carpoolers
r	Lane types: HOT lane - H, GP lane - G
S_r	Bottleneck flow capacity on lane r
α_s	Travel cost parameter (value of time)
β_s	Early arrival penalty parameter
γ_s	Late arrival penalty parameter
τ_r^0	Free-flow travel time on lane r
τ_r^w	Shockwave travel time on lane r
Time-varying variables	
$q_r(t)$	Queue length on lane r at time t
$p_r(t)$	Inflow rate on lane r at time t
$v_r(t)$	Exit flow rate on lane r at time t
$\tau_r(t)$	Travel time on lane r at time t
$c_r^s(t)$	Generalized cost for solo-drivers on lane r at time t
$c_H^l(t)$	Generalized cost for carpoolers on the HOT lane at time t
c_{min}^s	Minimal travel cost for solo-drivers in the two-equilibria model
c_{min}^l	Minimal travel cost for carpoolers in the two-equilibria model
c_{min}	Minimal travel cost for all commuters in the one-equilibrium model
$\Phi(t)$	Toll charge for solo-drivers on the HOT lane at time t
$\Psi(t)$	Inconvenience cost for carpollers at time t
Intermediate notations	
t^*	Preferred arrival time (without early/late arrival penalty)
t'	The earliest departure time
t''	The latest departure time
T	Terminal time

2.2 Problem Description

For the dynamic morning commute problem, we assume a corridor connecting home and the central business district (CBD) as shown in Figure 1, where home is considered as the origin and CBD is considered as the destination. This corridor is divided into GP lanes G and HOT lanes H with the maximum throughput limit (S_H, S_G) (bottleneck capacity). The inflow generated at home is diverged into HOT flow and GP flow, heading to CBD, which could form a queue at the bottleneck. There is a lane capacity ($\bar{Q}_r$) to represent the maximum queue for each lane. The flow is composed of solo-drivers and carpoolers. For solo-drivers, each individual can drive using the GP lane for free or pay a toll to use a HOT lane, while carpoolers only choose the HOT lane. In this study, we analyze the traffic performance under the user equilibrium condition.

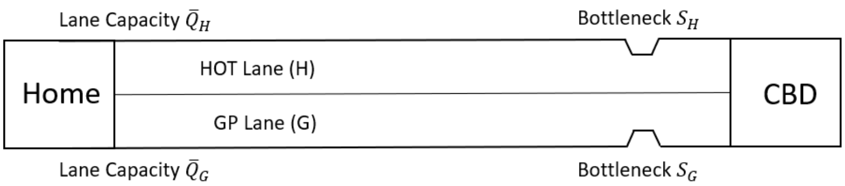

Figure 1: Graphical representation of the corridor

2.3 Point Queue Dynamics

We first propose a continuous-time point queue model at the link level, which will be formulated as a linear complementary system (LCS) alternatively in the following sections. Like Ban et al. (2012a), this is a queue model where no spillbacks are involved. The dynamics of a lane $r \in (H, G)$ are described as follows:

$$\dot{q}_r(t) = p_r(t - \tau_r^0) - v_r(t)$$

$$v_r(t) = \begin{cases} p_r(t - \tau_r^0), & \text{if } q_r(t) = 0 \text{ and } p_r(t - \tau_r^0) < S_r \\ S_r, & \text{otherwise} \end{cases}$$

where $p_r(t)$ represents the inflow of lane r and $v_r(t)$ represents the exit flow of lane r. $q_r(t)$ represents the queue length on lane r. Also, the travel time τ for lane r is calculated as:

$$\tau_r(t) = \tau_r^0 + \frac{q_r(t + \tau_r^0)}{S_r}$$

where τ_r^0 is the free-flow travel time on lane r. Note that the above Point Queue (PQ) model performs well enough when spillbacks do not occur, which is the case considered in this study (it is equivalent to have a long road such that the queue length can go to infinity).

2.4 Dynamic User Equilibrium and Commuters' Choice

To complete the DUE model, we need to capture the travelers' choice behavior while using the HOT lane system. In this section, departure-time choices are incorporated with an early or late arrival penalty such that commuters choose a departure time, a lane to use and/or a travel mode to minimize their travel costs. We conducted an user equilibrium analysis based on the travelers' cost function c. Note that the solo-drivers would potentially pay a toll $\Phi(t) \geq 0$ to use the HOT lane while carpoolers can use the HOT lane free of charge, but they will incur an inconvenience cost $\Psi(t) \geq 0$, which includes possible payment to the carpool company, time for picking up or waiting, anxiety to ride with strangers, extra fuel cost, etc. Denote $c^s_G(t)$ as the generalized travel cost for solo-drivers on a general lane at time t, $c^s_r(t)$ as the generalized travel cost for solo-drivers on lane $r \in \{H, G\}$ at time t. Then the generalized travel cost for each type of commuter is as follows:

$$c^s_H(t) = \alpha_1 \tau_H(t) + max[\gamma_1(t + \tau_H(t) - t^*), \beta_1(t^* - t - \tau_H(t))] + \Phi(t)$$

$$c^s_G(t) = \alpha_2 \tau_G(t) + max[\gamma_2(t + \tau_G(t) - t^*), \beta_2(t^* - t - \tau_G(t))]$$

$$c^l_H(t) = \alpha_3 \tau_H(t) + max[\gamma_3(t + \tau_H(t) - t^*), \beta_3(t^* - t - \tau_H(t))] + \Psi(t)$$

Here $\tau_H(t)$ and $\tau_G(t)$ are the total travel time for each type of traveler on each type of lane. Here $\Phi(t)$ denotes the toll charge for solo-drivers on the HOT lane and $\Psi(t)$ denotes the inconvenience cost of the carpoolers. For the solo-drivers who pay for a toll charge, since each vehicle only has one solo-driver, the vehicle cost is equivalent to $c^s_H(t)$. As for the carpoolers, $c^l_H(t)$ consists of the travel time cost, early arrival or late arrival penalty, and inconvenience cost. We assume that the inconvenience cost is shareable among the driver and riders in the same carpool vehicle. For example, carpool driver and riders will share the price of fuel and parking. As a result, the travel costs of the carpool driver and carpool riders are the same in each vehicle, all equal to $c^l_H(t)$. Thus, in our model, the vehicle costs are equivalent to the commuter costs for both solo-drivers and carpoolers.

Denote the minimum cost of solo-drivers as c^s_{min} and the minimum cost of carpoolers as c^l_{min}. A commuter will choose to depart at a time when (s)he has the smallest travel cost, and at the same time choose a lane with the smallest travel cost. For example, carpoolers will choose to depart at a time t when $c^l_H(t) = c^l_{min}$, and no carpoolers will depart at a time t when $c^l_H(t) > c^l_{min}$; similarly, solo-drivers will choose to drive on the HOT lane if $c^s_H(t) = c^s_{min}$, and none of the solo-drivers will travel on the GP lane when $c^s_G(t) > c^s_{min}$. Thus, we have the following complementarity conditions to determine the departure time choice:

$$0 \le p_H^s(t) \perp (c_H^s(t) - c_{min}^s) \ge 0$$

$$0 \le p_G^s(t) \perp (c_G^s(t) - c_{min}^s) \ge 0$$

$$0 \le p_H^l(t) \perp (c_H^l(t) - c_{min}^l) \ge 0$$

When the commuters reach one equilibrium, namely $c_{min} \triangleq c_{min}^s = c_{min}^l$, we can also include mode choice in the model. In other words, commuters will only choose to be solo-drivers on the HOT lane when $c_H^s(t) = c_{min}$, $c_G^s(t) > c_{min}$ and $c_H^l(t) > c_{min}$. Thus we have,

$$0 \le p_H^s(t) \perp (c_H^s(t) - c_{min}) \ge 0$$

$$0 \le p_G^s(t) \perp (c_G^s(t) - c_{min}) \ge 0$$

$$0 \le p_H^l(t) \perp (c_H^l(t) - c_{min}) \ge 0$$

2.5 The Overall Differential Complementarity System Models

2.5.1 Morning Commute Problem with HOT Lanes: Two Equilibria

In this section, we consider the model in which there are two equilibria in the system: solo-drivers on the HOT lane and solo-drivers on the GP lane reach one equilibrium, while carpoolers reach the other equilibrium. The total demand of solo-drivers and total demand of carpoolers are known and fixed, and solo-drivers can choose to drive on a HOT lane or a GP lane.

Based on the above analysis, the two equilibria model can be summarized as follows:

(a) Point Queue Dynamics:

The point queue dynamics are as follows:

$$\dot{q}_r(t) = p_r(t - \tau_r^0) - v_r(t) \quad \forall r \in \{H, G\} \tag{1}$$

And the link travel time is calculated as:

$$\tau_r(t) = \tau_r^0 + \frac{q_r(t + \tau_r^0)}{S_r} \quad \forall r \in \{H, G\} \tag{2}$$

(b) DUE and Departure Time Choice:

$$
\begin{aligned}
&0 \leq p_H^s(t) \perp (c_H^s(t) - c_{min}^s) \geq 0 \\
&0 \leq p_G^s(t) \perp (c_G^s(t) - c_{min}^s) \geq 0 \\
&0 \leq p_H^l(t) \perp (c_H^l(t) - c_{min}^l) \geq 0
\end{aligned} \tag{3}
$$

(c) Node Model

- Flow conservation:

$$
\begin{aligned}
&p_H(t) = p_H^l(t) + p_H^s(t) \\
&p_G(t) = p_G^s(t)
\end{aligned} \tag{4}
$$

- Nonnegativity and capcity constraints:

$$
\begin{aligned}
&0 \leq q_r(t) \perp \zeta_r(t) \geq 0 \quad \forall r \in \{H, G\} \\
&v_r(t) = S_r - \zeta_r(t) \qquad \forall r \in \{H, G\}
\end{aligned} \tag{5}
$$

Here $v_r(t)$ is guaranteed to be nonnegative by Ban et al. (2012b).

Initial and Terminal Conditions

- Initial queues:

$$q_r(0) = 0 \quad \forall r \in \{H, G\} \tag{6}$$

- Terminal queues:

$$q_r(T) = 0 \quad \forall r \in \{H, G\} \tag{7}$$

- Cumulative flows for demand conservation:

$$
\begin{aligned}
&\int_0^T p_H^l(t)\, dt = D^l \\
&\int_0^T (p_H^s(t) + p_G^s(t))\, dt = D^s
\end{aligned} \tag{8}
$$

2.5.2 Morning Commute Problem with HOT Lanes: One Equilibrium

In this section, we consider the model in which there is one equilibrium in the system: solo-drivers on the HOT lane, solo-drivers on the GP lane and carpoolers have identical travel cost under a single equilibrium. In this case, only the total demand of travelers is known, the travelers can choose to be carpoolers, solo-drivers on the HOT lane or solo-drivers on the GP lane.

Following the same procedure as the two-equilibria case, the one-equilibrium model can be summarized as follows:

(a) Point Queue Dynamics: Equations (1 - 2)

(b) DUE and Departure Time Choice:

$$
\begin{aligned}
&0 \leq p_H^s(t) \perp (c_H^s(t) - c_{min}) \geq 0 \\
&0 \leq p_G^s(t) \perp (c_G^s(t) - c_{min}) \geq 0 \\
&0 \leq p_H^l(t) \perp (c_H^l(t) - c_{min}) \geq 0
\end{aligned}
\tag{9}
$$

(c) Node Model

- Flow conservation: Equation (4)
- Nonnegativity and capcity constraints: Equation (5)

Initial and Terminal Conditions

- Initial queues: Equation (6)
- Terminal queues: Equation (7)
- Cumulative flows for demand conservation:

$$
\int_0^T \left(p_H^l(t) + p_H^s(t) + p_G^s(t)\right)\, dt = D \tag{10}
$$

2.6 Discretization of the Proposed Models

In this section, we convert the proposed continuous-time DCS models into discrete-time subproblems. The approach we use is similar to that of Pang et al. (2012) and Ma et al. (2018).

Given the step size h, we divide the time interval $[0, T]$ into N subintervals as follows:

$$0 \triangleq t_{h;0} \leq t_{h;1} \leq t_{h;2} \leq \cdots \leq t_{h;N-1} \leq t_{h;N} \triangleq T$$

Notably, $t_{h;N} \triangleq t_{h;N-1} + h$. Let $n_r^{0;h} \triangleq \tau_r^0/h$, and $n_r^{\omega;h} \triangleq \tau_r^{\omega}/h$, which represent the discretizations of the free flow travel time and shockwave travel time, respectively. Then the discrete-time problems of the two DCS models can be written as in the following two sections.

2.6.1 Discretization of the Two-Equilibria Model

(a) Point Queue Dynamics:

The point queue dynamics is as follows:

$$\begin{aligned}
&q_r^{h;i} = 0 \quad \forall i = 0, 1, \ldots, n_r^{0;h}; \quad \forall r \in \{H, G\} \\
&\tfrac{q_r^{h;i} - q_r^{h;i-1}}{h} = p_r^{h;i-n_r^{0;h}} - v_r^{h;i} \quad \forall i = n_r^{0;h} + 1, \ldots, N; \quad \forall r \in \{H, G\} \\
&q_r^{h;N} = 0 \qquad \forall r \in \{H, G\}
\end{aligned} \tag{11}$$

and the link travel time is calculated as:

$$\tau_r^{h;i} = \begin{cases} \tau_r^{0;h} & \forall i = 0, 1, \ldots, n_r^{0;h}; \quad \forall r \in \{H, G\} \\ \tau_r^{0;h} + \dfrac{q_r^{h;(i+n_r^{0;h})}}{S_r} & \forall i = n_r^{0;h} + 1, \ldots, N - n_r^{0;h}; \quad \forall r \in \{H, G\} \end{cases} \tag{12}$$

To ensure the cumulative flow is equal to the total demand, we need:

$$\begin{aligned}
&\textstyle\sum_{i=0}^{N} (p_H^{s;h;i} + p_G^{s;h;i}) = D^s \\
&\textstyle\sum_{i=0}^{N} p_H^{l;h;i} = D^l
\end{aligned} \tag{13}$$

(b) DUE and Departure Time Choice:

$$\begin{aligned}
&0 \leq p_H^{s;h;i} \perp (c_H^{s;h;i} - c_{min}^s) \geq 0 \quad \forall i = 0, 1, \ldots, N - n_H^{0;h} \\
&0 \leq p_G^{s;h;i} \perp (c_G^{s;h;i} - c_{min}^s) \geq 0 \quad \forall i = 0, 1, \ldots, N - n_G^{0;h} \\
&0 \leq p_H^{l;h;i} \perp (c_H^{l;h;i} - c_{min}^l) \geq 0 \quad \forall i = 0, 1, \ldots, N - n_H^{0;h}
\end{aligned} \tag{14}$$

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where the generalized travel costs for each type of commuter are defined as:

$$
\begin{aligned}
c_H^{s;h;i} &= \alpha_1 \tau_H^{h;i} + max[\gamma_1(i \times h - t^* + \tau_H^{h;i}), \beta_1(t^* - i \times h - \tau_H^{h;i})] + \Phi^{h;i} \quad \forall i = 0, 1, \ldots, N - n_H^{0;h} \\
c_G^{s;h;i} &= \alpha_2 \tau_G^{h;i} + max[\gamma_2(i \times h - t^* + \tau_G^{h;i}), \beta_2(t^* - i \times h - \tau_G^{h;i}))] \quad \forall i = 0, 1, \ldots, N - n_G^{0;h} \\
c_H^{l;h;i} &= \alpha_3 \tau_H^{h;i} + max[\gamma_3(i \times h - t^* + \tau_H^{h;i}), \beta_3(t^* - i \times h - \tau_H^{h;i})] + \Psi^{h;i} \quad \forall i = 0, 1, \ldots, N - n_H^{0;h}
\end{aligned} \tag{15}
$$

(c) Node Model

- Flow conservation:

$$
\begin{aligned}
p_H^{h;i} &= p_H^{l;h;i} + p_H^{s;h;i} \quad \forall i = 0, 1, \ldots, N - n_H^{0;h} \\
p_G^{h;i} &= p_G^{s;h;i} \quad \forall i = 0, 1, \ldots, N - n_G^{0;h}
\end{aligned} \tag{16}
$$

with $p_H^{h;i} = p_H^{l;h;i} = p_H^{s;h;i} = 0$ for $i = N - n_H^{0;h} + 1, \ldots, N$ and $p_G^{h;i} = p_G^{s;h;i} = 0$ for $i = N - n_G^{0;h} + 1, \ldots, N$.

- Nonnegativity and capcity constraints:

$$
0 \leq q_r^{h;i} \perp \zeta_r^{h;i} \geq 0 \quad \forall i = n_r^{0;h} + 1, \ldots, N; \forall r \in \{H, G\} \tag{17}
$$

with $\zeta_r^{h;i} = 0$ for $i = 0, 1, \ldots, n_r^{0;h}$ and $r \in \{H, G\}$.

$$
v_r^{h;i} = S_r - \zeta_r^{h;i} \quad \forall i = n_r^{0;h} + 1, \ldots, N; \forall r \in \{H, G\} \tag{18}
$$

with $v_r^{h;i} = 0$ for $i = 0, 1, \ldots, n_r^{0;h}$ and $r \in \{H, G\}$.

2.6.2 Discretization of the One-Equilibrium Model

(a) Point Queue Dynamics:

The point queue dynamics: Equations (11 - 12)

To ensure the cumulative flow is equal to the total demand, we need:

$$
\sum_{i=0}^{N} (p_H^{s;h;i} + p_G^{s;h;i} + p_H^{l;h;i}) = D \tag{19}
$$

(b) DUE and Departure Time Choice:

$$
\begin{aligned}
&0 \le p_H^{s;h;i} \perp (c_H^{s;h;i} - c_{min}) \ge 0 \quad \forall i = 0, 1, \ldots, N - n_H^{0;h} \\
&0 \le p_G^{s;h;i} \perp (c_G^{s;h;i} - c_{min}) \ge 0 \quad \forall i = 0, 1, \ldots, N - n_G^{0;h} \\
&0 \le p_H^{l;h;i} \perp (c_H^{l;h;i} - c_{min}) \ge 0 \quad \forall i = 0, 1, \ldots, N - n_H^{0;h}
\end{aligned}
\tag{20}
$$

where the generalized travel costs for each type of commuter are defined as in Equation (15).

(c) Node Model

- Flow conservation: Equation (16)
- Nonnegativity and capcity constraints: Equations (17 - 18)

3 Analysis of Inconvenience Costs and Toll Charges

In this section, our main goal is to quantify the inconvenience costs for carpoolers under an equilibrium condition and decide the toll charges for solo-drivers to (1) maintain the HOT lane in a congestion-free condition, which benefits all travelers and (2) encourage solo-drivers to pay for using the HOT lane, without generating an extra cost for carpoolers. To achieve these goals, two analytical models have been developed: one is the time-variant inconvenience costs for the two-equilibria model, and another is the time-variant inconvenience costs and dynamic toll charges for the one-equilibrium model.

3.1 Time-variant Inconvenience Costs for the Two-Equilibria Model

In the two equilibria situation, carpoolers on a HOT lane reach their own equilibrium. Here we analyse the equilibrium of carpoolers on the HOT lane to quantify the inconvenience costs under an equilibrium condition. For simplicity, we use α, β, γ to denote α_3, β_3, γ_3 respectively in the following analysis. When DUE is achieved, the travelers must have identical travel cost.

Denote t' as the first departure time, t'' as the last departure time, $\hat{t}$ as the departure time without early/late arrival penalty, namely $\hat{t} = t^* - \tau_H(t)$ (in the free flow condition $\hat{t} = t^* - \tau_H^0$). In order to have some space for solo-drivers on the HOT lane, reserved capacity for solo-drivers on the HOT lane is set as a model input, notated as μ. Let N_H denote the total number of carpool vehicles travelling on the HOT lane, S_H denote the bottleneck capacity for the HOT lane. Under the DUE condition, we have $c_H^l(t') = c_H^l(t'')$. Together with the flow conservation on the HOT lane from time t' to time t'' we have,

$$\begin{cases} \text{DUE condition} \Rightarrow \alpha\tau_0 + \beta(t^* - t' - \tau_0) + \Psi(t') = \alpha\tau_0 + \gamma(t'' + \tau_0 - t^*) + \Psi(t'') \\ \text{Flow conservation} \Rightarrow (t'' + \tau_0) - (t' + \tau_0) = \frac{N_H}{(1-\mu)\,S_H} \end{cases}$$

Then we have,

$$\begin{cases} t'' = t^* - \tau_0 + \frac{[\Psi(t') - \Psi(t'')]}{\beta+\gamma} + \frac{N_H}{(1-\mu)\,S_H}\frac{\beta}{\beta+\gamma} \\ t' = t^* - \tau_0 + \frac{[\Psi(t') - \Psi(t'')]}{\beta+\gamma} - \frac{N_H}{(1-\mu)\,S_H}\frac{\gamma}{\beta+\gamma} \end{cases}$$

Thus, under the equilibrium condition, the generalized cost for carpoolers can be calculated as follows

$$c^l_{min} = c^l_H(t') = c^l_H(t'') = \alpha\tau_0 + \frac{\beta\gamma}{\beta+\gamma}\frac{N_H}{(1-\mu)\,S_H} + \frac{1}{\beta+\gamma}[\gamma\Psi(t') + \beta\Psi(t'')]$$

Recall the generalized cost function:

$$c^l_H(t) = \begin{cases} \alpha\tau_H(t) + \beta(t^* - t - \tau_H(t)) + \Psi(t) & t \in (t', \hat{t}) \\ \alpha\tau_H(t) + \gamma(t + \tau_H(t) - t^*) + \Psi(t) & t \in (\hat{t}, t'') \end{cases}$$

Under DUE we have $\frac{dc^l_H(t)}{dt} = 0$, which leads to:

$$\begin{cases} \alpha\dot{\tau_H}(t) + \beta(-1 - \dot{\tau_H}(t)) + \dot{\Psi}(t) = 0 & t \in (t', \hat{t}) \\ \alpha\dot{\tau_H}(t) + \gamma(1 + \dot{\tau_H}(t)) + \dot{\Psi}(t) = 0 & t \in (\hat{t}, t'') \end{cases}$$

With Equation (1 - 2), for the first equation when $t \in (t', \hat{t})$

$$(\alpha - \beta)[\frac{p(t - \tau^0_H) - v(t)}{(1-\mu)\,S_H}] = \beta - \dot{\Psi}(t)$$

In the situation when there is no queueing on a HOT lane, we have $p(t - \tau^0_H) = v(t)$. As a result, $\dot{\Psi}(t) = \beta$:

$$\Psi(t) = \beta t + \Delta_1 \quad t \in (t', \hat{t}) \tag{21}$$

Following a similar process as above, we can obtain

$$\Psi(t) = \Delta_2 - \gamma t \quad t \in (\hat{t}, t'') \tag{22}$$

Here in Equations (21 - 22), Δ_1 and Δ_2 are two constants to be determined.

Without loss of generality, we set the inconvenience cost at time t' and inconvenience cost at time t'' to be equal, namely $\Psi(t') = \Psi(t'')$. Then from

$$\begin{cases} \Psi(\hat{t}) = \beta\hat{t} + \Delta_1 = \Delta_2 - \gamma\hat{t} \\ \Psi(t') = \beta t' + \Delta_1 = \Psi(t'') = \Delta_2 - \gamma t'' \end{cases}$$

we have

$$\begin{cases} \Delta_1 = -\beta t' \\ \Delta_2 = \gamma t'' \end{cases}$$

Thus, $\Psi(t)$ can be written as a piecewise linear function as follows:

$$\Psi(t) = \begin{cases} 0 & t \in (0, t') \\ \beta(t - t') & t \in (t', \hat{t}) \\ \gamma(t'' - t) & t \in (\hat{t}, t'') \\ 0 & t \in (t'', T) \end{cases} \tag{23}$$

where

$$\begin{cases} t'' = t^* - \tau_0 + \frac{N_H}{(1-\mu)\,S_H}\frac{\beta}{\beta+\gamma} \\ t' = t^* - \tau_0 - \frac{N_H}{(1-\mu)\,S_H}\frac{\gamma}{\beta+\gamma} \end{cases}$$

Here we can guarantee that $\Psi(t) \geq 0$, namely the inconvenience costs are nonnegative. And $\Psi(t') = \Psi(t'') = 0$, which means that the inconvenience costs at time t' and time t'' are zero.

Recall that under an equilibrium condition, the generalized cost for carpoolers is as follows:

$$c_{min} = \alpha\tau_0 + \frac{\beta\gamma}{\beta+\gamma}\frac{N_H}{(1-\mu)\,S_H} + \frac{1}{\beta+\gamma}[\gamma\Psi(t') + \beta\Psi(t'')] \geq \alpha\tau_0 + \frac{\beta\gamma}{\beta+\gamma}\frac{N_H}{(1-\mu)\,S_H}$$

Our analysis shows that, in the equilibrium context, if we want to avoid queueing for carpoolers on the HOT lane, and keep carpoolers' generalized cost at a minimum, the inconvenience cost should follow Equation (23). Otherwise, carpoolers will not reach an equilibrium.

3.2 Dynamic Toll Charges and Time-variant Inconvenience Costs for the One-Equilibrium Model

Under DUE conditions, we have all commuters share the same minimized cost:

$$c_H^l(t) = c_G^s(t) = c_H^s(t) = c_{min}, \quad \forall t$$

Since solo-drivers and carpoolers are mixed on the HOT lane, we first explore the relationship between the inconvenience costs and toll charges under an one equilibrium condition. Specifically, we want to inspect the HOT lane related generalized costs: $c^s_H(t)$ and $c^l_H(t)$. When reaching an equilibrium, and both carpoolers and solo-drivers exist on the HOT lane, we have $c^s_H(t) = c^l_H(t) = c_{min}$. Thus, we can derive the relationship between $\Phi(t)$ and $\Psi(t)$:

$$(\alpha_3 - \alpha_1)\tau_0 = \Phi(t) - \Psi(t)$$

Assume they share the same t' and t'', meaning the two types of lanes share the same time window for the morning commute problem.

$$t'_G = t'_H \quad t''_G = t''_H$$

Then, we have the following relation:

$$(t'' - t')S_H = N_H$$

$$(t'' - t')S_G = N_G$$

and get:

$$\frac{N_H}{S_H} = \frac{N_G}{S_G}$$

Since the total number of vehicles is fixed:

$$N = N_H + N_G = (t'' - t')(S_H + S_G)$$

Recall the minimized cost for c^l_{min} and c^s_{min}:

$$c^l_{min} = \alpha_3\tau_0 + \frac{\beta\gamma}{\beta+\gamma}\frac{N_H}{S_H} + \frac{1}{\beta+\gamma}[\gamma\Psi(t') + \beta\Psi(t'')]$$

$$c^s_{min} = \alpha_2\tau_0 + \frac{\beta\gamma}{\beta+\gamma}\frac{N_G}{S_G}$$

Since they share the same minimized cost, we have $c^l_{min} = c^s_{min}$. Thus, we get:

$$(\beta+\gamma)(\alpha_2 - \alpha_3)\tau_0 = \beta\gamma(\frac{N_H}{S_H} - \frac{N_G}{S_G}) + [\gamma\Psi(t') + \beta\Psi(t'')]$$

Under the DUE condition, it can be simplified as:

$$(\beta+\gamma)(\alpha_2-\alpha_3)\tau_0 = [\gamma\Psi(t') + \beta\Psi(t'')]$$

With the pricing condition in the first section:

$$\begin{cases} \gamma\Psi(t') = \gamma\Delta \\ \beta\Psi(t'') = -\beta\gamma(t''-\hat{t}) + \beta\Delta + \beta^2(\hat{t}-t') \end{cases}$$

We plug in $\Psi(t')$ and $\Psi(t'')$ into the minimized cost equation:

$$(\alpha_2-\alpha_3)\tau_0 + \frac{\beta}{\beta+\gamma}(\gamma t'' + \beta t') = \Delta + \beta\hat{t}$$

The cost at time t' for both types of travellers is:

$$\begin{cases} c^s(t') = \alpha_2\tau_0 + \beta(\hat{t}-t') \\ c^l(t') = \alpha_3\tau_0 + \beta(\hat{t}-t') + \Delta \end{cases}$$

Since they share the same cost $c^s(t') = c^l(t')$, we get:

$$\Delta = (\alpha_2-\alpha_3)\tau_0$$

Recall from Equation (21), we showed that the change rate of inconvenience cost is equal to β. Hence, the beginning and end time of the morning commute problem is:

$$\begin{cases} t' = \hat{t} - \frac{\gamma N}{(S_H+S_G)(\beta+\gamma)} \\ t'' = \frac{N}{(S_H+S_G)} + t' \end{cases}$$

Thus, the inconvenience costs and the toll charges can be derived in closed-form formulas as follows:

$$\Psi(t) = \begin{cases} 0 & t \in (0, t') \\ \beta(t - t') + (\alpha_2 - \alpha_3)\tau_0 & t \in (t', \hat{t}) \\ \beta(\hat{t} - t') + (\alpha_2 - \alpha_3)\tau_0 - \gamma(t - \hat{t}) & t \in (\hat{t}, t'') \\ 0 & t \in (t'', T) \end{cases}$$
$$\Phi(t) = \begin{cases} 0 & t \in (0, t') \\ \beta(t - t') + (\alpha_2 - \alpha_1)\tau_0 & t \in (t', \hat{t}) \\ \beta(\hat{t} - t') + (\alpha_2 - \alpha_1)\tau_0 - \gamma(t - \hat{t}) & t \in (\hat{t}, t'') \\ 0 & t \in (t'', T) \end{cases} \tag{24}$$

4 Numerical Experiments

In this section, we conduct numerial experiments to illustrate how the proposed DCS models and analytical results can be used by decision makers to make policy. The discretized DCS models are solved by the Knitro Solver.

We choose a 25-mile corridor without ramps. Different scenarios are considered for demonstration of HOT lanes' benefits. There are 4 lanes in total. In section 4.1, we consider the scenario that includes one HOT lane and three GP lanes: (1) we show that the queueing on the HOT lane can be avoided if we set the proper toll charge through sensitivity analysis, and the time-variant inconvenience costs of carpoolers follow the conditions as derived in Section 3.1 under the equilibrium condition; (2) we compare the proposed two-equilibria model with a model without tolling, and the results show the advantages of our model in terms of system performance such as queue length and vehicle hours traveled (VHT); (3) we set different reserved capacity for solo-drivers on a HOT lane, and compare the relevant system performance. In Section 4.2, we consider two additional scenarios that convert GP lanes into HOT lanes and compare them with the base scenario (one HOT lane and three GP lanes): (1) two HOT lanes and two GP lanes; (2) three HOT lanes and one GP lane.

For the α's, β's and γ's, we follow Arnott et al. (1987) and assign different commuter types with different α values but same β and γ. Here we assume that $\gamma > \alpha_1 = \alpha_2 > \alpha_3 > \beta$. This assumption is reasonable because commuters with large/small early penalties are more likely to have large/small late penalties. The values of the model parameters are as follows:

α_1:	\$24/h
α_2:	\$24/h
α_3:	\$21/h
β:	\$12/h
γ:	\$48/h
t^*:	8 A.M.
Time horizon:	7 A.M. - 9 A.M.
Time step:	1 min
S_H: 1H/2H/3H	1600/4000/6000 vehs/h
S_G: 1GP/2GP/3GP	2000/4000/6000 vehs/h
τ_r^0:	30 mins
D:	6400 vehs
D^s:	5260 vehs
D^l:	1140 vehs
Average number of travelers per carpool vehicle	2

For comparison, travel cost, VHT, delays and number of commuters are computed to show the performance of the proposed models under different parameter settings and scenarios. Note that the per lane HOT lane capacity increases with multiple HOT lanes. This is based on observations from field data, which is shown in the Appendix.

4.1 Results of the Two-Equilibria Model

In the two-equilibria model, the total number of solo-drivers and carpoolers are known and fixed. All commuters can choose when to depart, and solo-drivers can choose to drive on HOT lanes or GP lanes. Carpoolers reach their own equilibrium on HOT lanes, while solo-drivers on HOT lanes and solo-drivers on GP lanes reach another equilibrium. In this section, the results are based on the scenario of one HOT lane and three GP lanes. Here we are trying to determine, when the number of HOT lanes and GP lanes are fixed, the resulting traffic patterns, and how the changes in the model parameters will affect the travellers in the system through:

- Sensitivity analysis to determine the congestion-free HOT lane toll charge
- Comparison with a no HOT lane tolling scenario
- Change in reserved capacity for solo-drivers on HOT lane

4.1.1 Sensitivity Analysis to Determine the Toll Charge

In the two-equilibria model, carpoolers reach their own equailbrium on the HOT lane. Here the reserved capacity for solo-drivers on the HOT lane is set to be 10 %. The time-variant inconvenience costs for carpoolers on the HOT lane are shown in Figure 2, as derived in Section 3.1 under the equilibrium condition. As we can see, the time-variant inconvenience costs form a piece-linear function in terms of time: first increases linearly then decreases linearly; reaches the maximum of 7.2 dollars at 7:48. The duration of a non-zero inconvenience costs for carpoolers is 56 minutes.

Since there are two equilibria on the HOT lane, we cannot set the toll charge analytically, due to the complex interactions between solo-drivers on the HOT lane and solo-drivers on the GP lanes. As an alternative, we use sensitivity analysis to determine the toll charge of solo-drivers on HOT lane. The results are shown in Figure 3. As we can see, when we increase the toll charge, the maximum queue length on the HOT lane decreases. And there will be no queueing on the HOT lane if we set the toll charge to be larger than 8.4 dollars, which is 1.2 dollars higher than the maximum inconvenience cost (7.2 dollars).

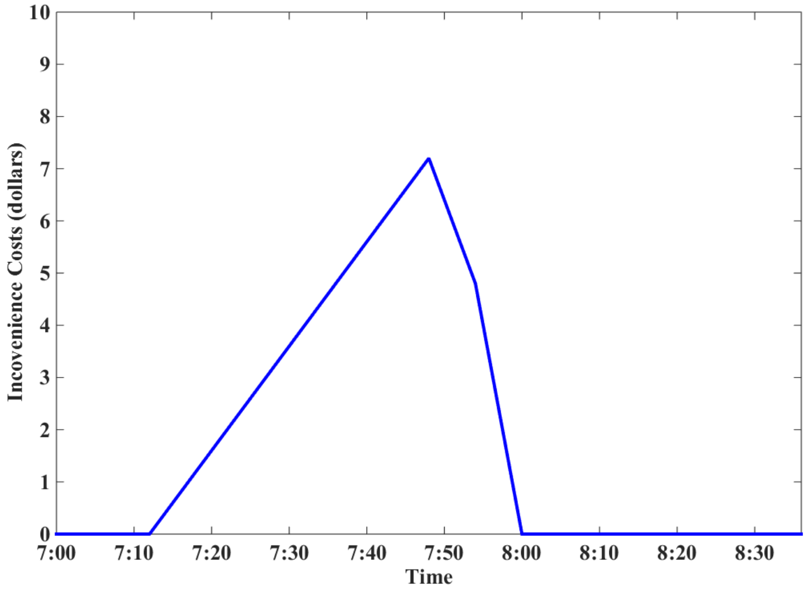

Figure 2: Time-variant Inconvenience Costs for Carpoolers on the HOT Lane under the Equilibrium Condition

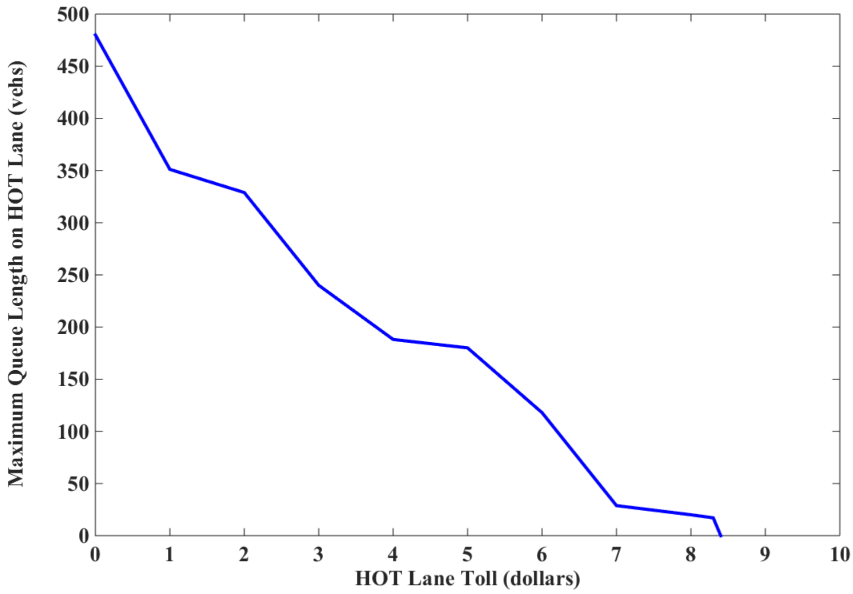

Figure 3: Sensitivity Analysis of Toll Charge and Maximum Queue Length on HOT Lane

4.1.2 Comparison with A No Tolling Scenario

In this section, we compare the proposed two-equilibria model with a model that has no HOT lane toll. The setting for the model without tolling is as follows: there are three GP lanes and one HOV lane. Carpoolers only drive on the HOV lane, while solo-drivers only drive on the GP lanes. There are two equilibria in this scenario, both carpoolers and solo-drivers reach their own equilibrium. And there is no tolling for both carpoolers and solo-drivers. This model can be solved by the existing literature such as Zhong et al. (2020).

The results of system performance before tolling and after tolling are listed in Table 1, which shows that in the model with tolling there are higher flows on the HOT lane, with no queueing, and lower total VHT and average commuter delay.

Table 1: Comparisons between Before Tolling and After Tolling

	Before Tolling	After Tolling
Maximum Queue Length on HOT Lane	426.7	0.0
# Solo-drivers on HOT Lane	0.0	140.1
# Solo-drivers on GP Lane	5260.0	5119.9
Total VHT	4445.2	4262.3
Average Commuter Delay	0.165	0.141

This better system performance with tolling can be explained by Figure 4 and Figure 5. Figure 4 and Figure 5 show the inflows and exit flows before tolling and after tolling, respectively. By comparing these two figures, we can notice that before tolling, many travelers tend to depart between 7:20 and 7:35 of the morning commute. Thus, the arrivals to the bottleneck is more concentrated, which leads to queueing on the HOT lane. While after tolling, the inflows of the HOT lane spread more evenly over time and are controlled not to exceed the HOT lane bottleneck capacity. As a result, there is no queueing on the HOT lane, and at the same time, it makes for a better use of the HOT lane capacity.

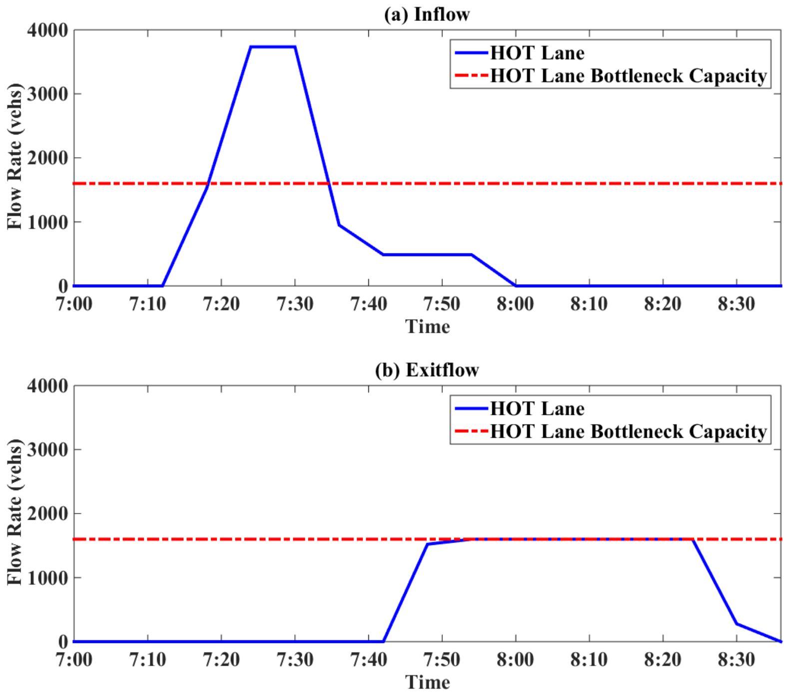

Figure 4: Inflows and Exit flows Before Tolling

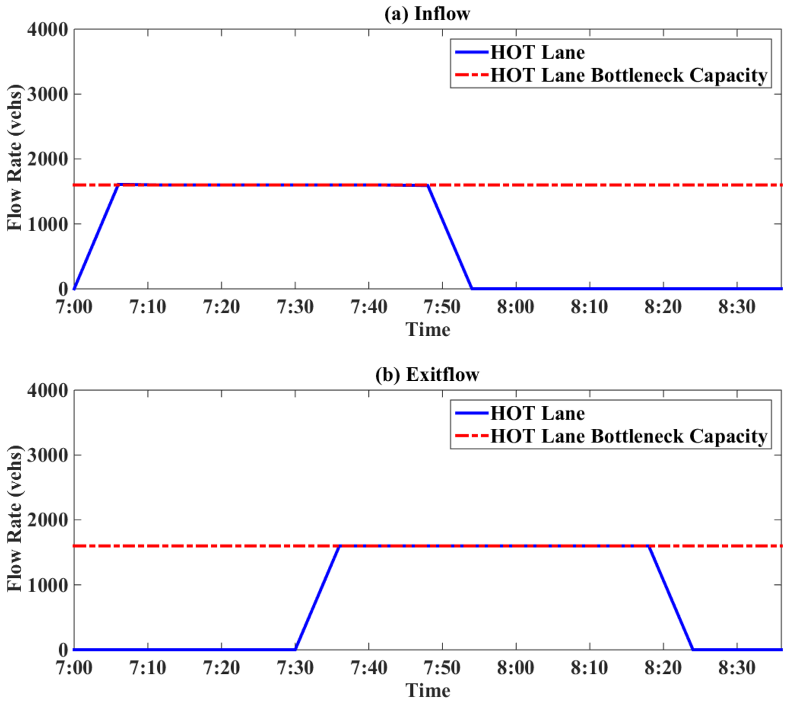

Figure 5: Inflows and Exit flows After Tolling

4.1.3 Influence of Reserved Capacity for Solo-drivers on HOT Lane

Recall that in Section 3.1, we reserve some capacity for solo-drivers so that they can also drive on the HOT lane. The reserved capacity is notated as μ. In this section, we analyze the influence of different reserved capacity values for solo-drivers on the HOT lane.

Figure 6 shows the difference of inconvenience costs for carpoolers compared to the base case (10% reserved capacity) under different reserved capacity values. When we reserve more capacity for solo-drivers on the HOT lane, the inconvenience costs turn out to be higher for the carpoolers, and the duration of experiencing non-zero charging inconvenience costs also becomes longer. As a result, the commuting cost for carpoolers increases when more HOT lane capacity is reserved for solo-drivers. For example, if we want to increase the reserved capacity from 10% to 20%, we need to increase the inconvenience costs of carpoolers between 7:00 and 7:53 by 2.4 dollars at maximum.

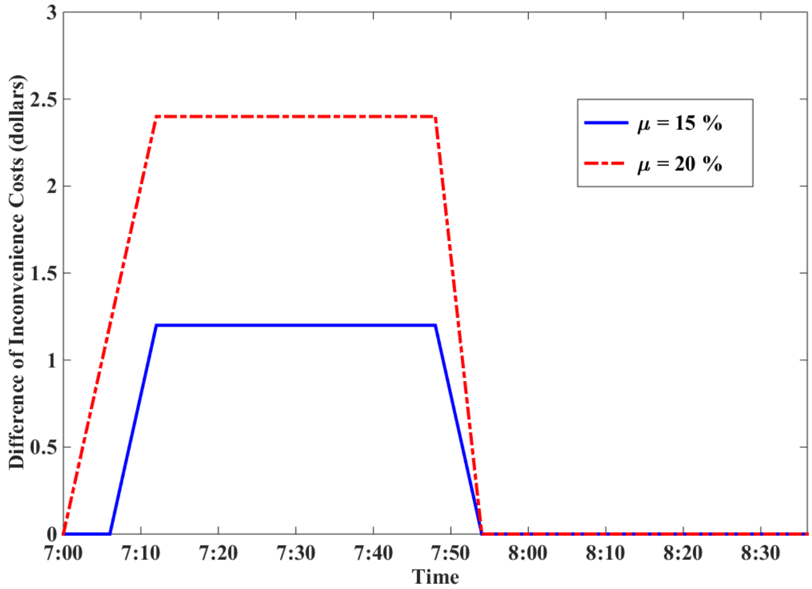

Figure 6: Difference of Inconvenience Costs for Carpoolers on the HOT Lane under Different Reserved Capacity Values compared to the Base Case ($\mu = 10\%$)

Then we compare the system performance under different reserved capacity values for solo-drivers on the HOT lane. The results are listed in Table 2. When we reserve more capacity for solo-drivers on the HOT lane, there will be more solo-drivers on the HOT lane, and the system performance improves: smaller total VHT, and smaller average commuter delay. The reason is that the GP lane is congested while the HOT lane is not in the tolled equilibrium. When we reserve more capacity for solo-drivers on the HOT lane, more solo-drivers will pay to use the HOT lane. Since the HOT lane is priced to be congestion-free, more commuters on it means less congestion on the GP lanes while causing no congestion to the HOT lane. At the same time, the commuting window (the time period when there are inflows) for HOT lane users become higher when more HOT lane capacity is reserved for solo-drivers, which means carpoolers will incur higher inconvenience costs and penalties although their travel time costs remain the same. As a result, the travel cost for carpoolers is increased when more capacity is reserved for solo-drivers. Although reserving more capacity for solo-drivers on the HOT lane reduces the total VHT and average commuter delay, the travel cost for carpoolers could be even higher than that of the solo-drivers when too much capacity is reserved for solo-drivers, as with the case of $\mu = 20\%$. This is clearly not desirable for promoting carpooling.

Table 2: Different reserved capacity for solo-drivers on HOT lane

Reserved Capacity for Solo-drivers on HOT Lane (μ)	10%	15%	20%
Commuting Window	7:12 - 7:54	7:06 - 7:54	7:00 - 7:54
Usage of the Reserved Capacity	100%	87%	78%
# Solo-drivers on HOT Lane	140.1	170.5	210.0
# Solo-drivers on GP Lane	5119.9	5089.5	5050.0
Total VHT	4262.3	4232.1	4134.4
Average Commuter Delay	0.141	0.137	0.124
Travel Cost of Carpoolers	18.9	19.0	20.1
Travel Cost of Solo-drivers	20.4	20.3	19.8

In addition, the HOT lane is not fully used under higher reserved capacity (Table 2). When we reserve more capacity for solo-drivers on the HOT lane, the usage of the reserved capacity decreases. The main reason is that the carpoolers and solo-drivers can have different equilibria, and a smaller portion of solo-drivers on the HOT lane could be enough to equalize the travel cost for solo-drivers on the HOT lane and the GP lanes.

4.2 Results of the One-Equilibrium Model

In the one-equilibrium model, travellers choose both their departure times and travel modes. All travelers reach the shared equilibrium on HOT and GP lanes and have the same generalized travel cost. Both the toll charges and inconvenience costs are controllable because for the former one, toll charges are naturally controllable and for the latter one, the inconvenience costs are impacted by the departure time choice of solo-drivers who are willing to pay to use the HOT lane and payment for matching drivers and riders. The dynamic toll charges and the time-variant inconvenience costs could be set based on the analysis in Section 3.2 to avoid queues on the HOT lane. Different from the two-equilibrium model, here we are interested in whether switching GP lanes to HOT lanes would benefit everyone in terms of the performance measures. The scenarios we studied are 1) the base case 1H3G: one HOT lane and three GP lanes; 2) the 2H2G case: two HOT lanes and two GP lanes, and 3) the 3H1G case: three HOT lanes and one GP lane. The results are presented in the subsequent sections.

4.2.1 Dynamic Toll Charges, Time-variant Inconvenience Costs, and Queue Length Performance

Before analyzing the results in detail, we first plot the dynamic toll charges and time-variant inconvenience costs in Figure 7. They are linearly dependent on time during the rush hour and the changing rate is consistent with α and β in the analysis Section of 3.2.

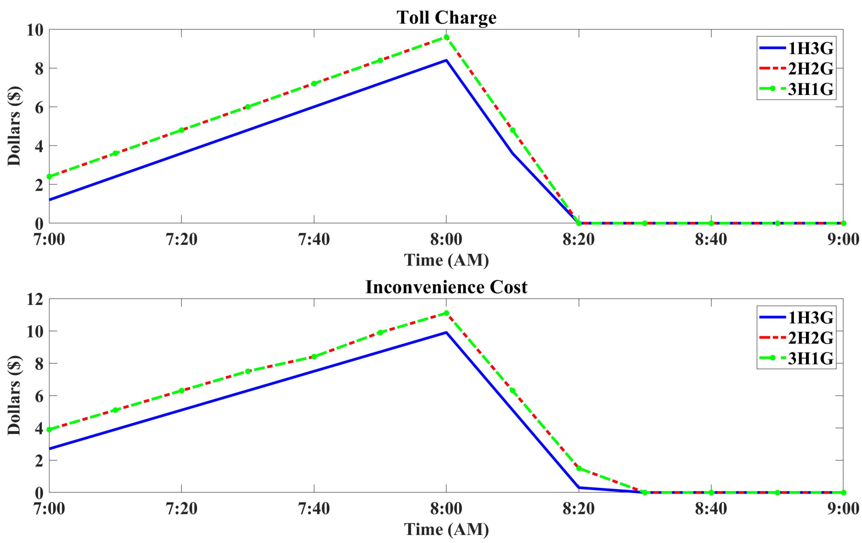

Figure 7: Toll Charges and Inconvenience Costs

Then we plot queues for the three scenarios in Figure 8. The queues on the HOT lane have been eliminated through dynamic toll charges no matter how many HOT lanes exist. The remaining queues in the first plot of Figure 8 are due to the numerical approximation errors. GP lanes reach their longest queues at the preferred arrival time t^*, which is consistent with Arnott's result (Arnott et al., 1987). We next explore the potential benefits by introducing more HOT lanes.

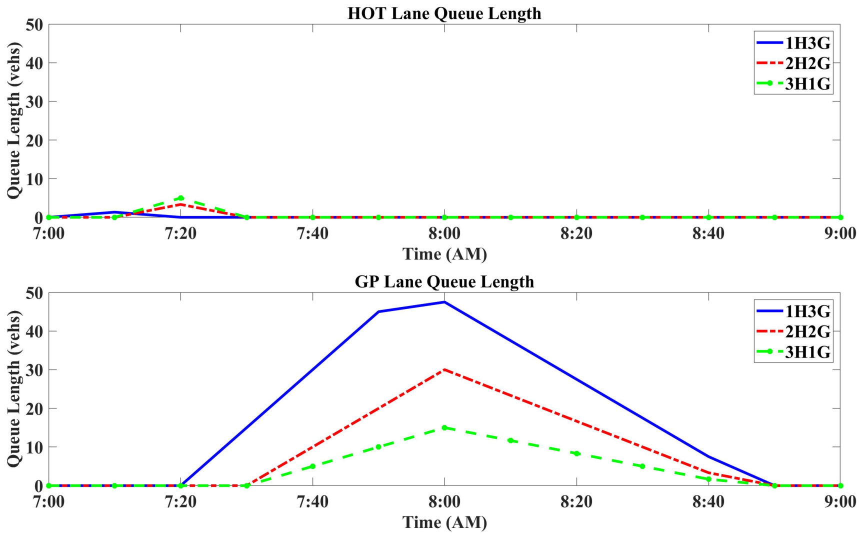

Figure 8: Queue Length on Each Lane

4.2.2 Modal Demand Performance

Table 3 shows the results with different number of HOT and GP lanes. Two points should be mentioned: 1) With more HOT lanes, travelers prefer traveling through them. 2) The amount of carpoolers increases with more HOT lanes. But the marginal effect is decreasing: When adding one additional HOT lane, the growth rate is nearly 150% . But the increment drops to 120% when there are 3 HOT lanes. The amount of solo-drivers on the HOT lane(s) varies: 463, 1298 and 1802. More HOT lanes attract more solo-drivers and the marginal effect decreases from a growth rate of 180% to 109%.

Table 3: Demand for each scenario (vehs)

	HOT	GP	Carpoolers	Solo-Drivers
1H3GP	1253	5127	810	5590
2H2GP	3320	3080	2022	4378
3H1GP	4795	1605	2993	3407

4.2.3 Cost Performance

Table 4 shows the overall and average cost for all scenarios. The travel cost of each traveler decreases due to the capacity change of multiple HOT lanes, which benefits both carpoolers and solo-drivers. Also, since more travelers choose to be carpoolers from Table 3, the total number of travelers going through the corridor increases accordingly.

Table 4: Cost for each scenario ($)

	Travel Cost of Each Commuter	Commuters Served	Total Travel Cost
1H3GP	20.4	7210	147084
2H2GP	19.2	8422	161702.4
3H1GP	19.2	9393	180345.6

4.2.4 VHT, VMT, and Delay Performance

Table 5 shows the vehicle hours traveled (VHT), vehicle miles traveled (VMT), and delay for the three cases, where SDs represent solo-drivers and CPs represent carpoolers. Note that the average delay is for each person, namely hrs/person. In general, the total VHT for solo-drivers decreases while that for carpoolers increases because more commuters choose carpooling, but the total VHT drops with more HOT lanes. This can be explained by the combined effects of more travelers choosing carpool, and more vehicles travel without congestion on the HOT lanes. The average delay also drops as more lanes are converted into HOT lanes, from 8.88 mins to 3.9 mins to 1.92 mins. For example, travelers only experience approximately 2 mins of delay with the case of three HOT lanes and one GP lane, a 78% reduction than the case of one HOT lane and three GP lanes. For the VMT, although the total VMT stays the same because of the fixed number of vehicles (16000), the average VMT for each traveler drops as more lanes are converted into HOT lanes. There is a 23% drop from the case of one HOT lane and three GP lanes to the case of three HOT lanes and one GP lane.

Table 5: VHT and delay for each scenario (hrs & miles)

	SDs VHT	CPs VHT	Total VHT	Total Delay	Average Delay	Commuters Served	VMT Per Commuter
1H3GP	3868.8	405	4268.8	1068.8	0.148	7210	22.2
2H2GP	2740.2	1011	3751.2	551.2	0.065	161702.4	19.0
3H1GP	2005.9	1496.5	3502.4	302.4	0.032	180345.6	17.0

5 Conclusions and Future Research

To understand how the HOT lanes can be used to benefit every traveler, we propose two morning commute DCS models that consider both solo-drivers and carpoolers with HOT and GP lanes. We treat commuters as cost minimizers who choose when to depart, either to drive on the HOT lane or GP lane, and/or either carpool or drive alone to minimize their generalized travel cost. This results in either a two-equilibria model (without mode choice) or an one-equilibrium model (with mode choice).

Under the two-equilibria scenario (no mode choice), we find:

- Congestion on HOT lanes can be eliminated by properly setting the toll charges.
- The proposed models outperforms a model without tolling, in terms of system performance such as VHT, and average commuter delay.
- Under our approach, solo-drivers are more willing to pay and drive on HOT lanes when they are underutilized by carpoolers.

Under the one-equilibrium scenario (with mode choice), we find:

- By considering time-variant inconvenience costs and dynamic tolling charges jointly, solo-drivers are willing to pay to use the HOT lane without impacting carpoolers, and HOT lanes remain congestion-free.
- Converting the GP lanes to HOT lanes not only attracts more solo-drivers to use it, but also increases the share of carpoolers. In total, the number of travelers on the HOT lanes increases.
- Tolling carpoolers does not increase the total cost. Instead, the average cost decreases, and the average delay and average VMT drop significantly.

The two-equilibria model decides the commuters' departure time choice and lane choice. This model can aid transportation planners and policy makers regulate the HOT lane usage when the historical data on the number of solo-drivers and carpoolers are known. The one-equilibrium model simultaneously decides the commuters' departure time choice, lane choice, and mode choice.

This study can be extended in several ways. For example, we can replace the point-queue bottleneck model with the double-queue model to capture the "spillover" effect. Another extension could be to consider the morning commute with HOT and GP lanes on a multi-origin multi-destination corridor. These extensions, however, lead to much less tractable problems.

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6 Data Management Plan

Products of Research

The main research products will be peer-reviewed journal articles, book chapters and/or conference proceedings targeted towards the transportation science research community, plus supplemental materials such as tables, numerical data used for graphs, etc. No personal data will be used in the project, so there is no threat of identity theft.

Data Format and Content

All research products will be available online in digital form. Manuscripts will appear in a common document-viewing format, such as PDF, and supplemental materials such as tables and numerical data will be in a tabular format, such as Microsoft Excel spreadsheet, tab-delimited text, etc.

Data Access and Sharing

All participants in the project will publish the results of their work. Papers will be published in peer-reviewed scientific journals, books published in English, conference proceedings, or as peer-reviewed data reports. Beyond the data posted on USC websites, primary data and other supporting materials created or gathered in the course of work will be shared with other researchers upon reasonable request, at no more than incremental cost and within a reasonable time of the request or, if later, the filing of a patent application covering the results of such research.

All the data used in the research are included in the table on Page 29 in the final report, which includes travel demand, bottleneck capacity, travel cost parameter (value of time), early arrival penalty parameter, late arrival penalty parameter, free-flow travel time, preferred arrival time (without early/late arrival penalty), etc.

Reuse and Redistribution

USC's policy is to encourage, wherever appropriate, research data to be shared with the general public through internet access. This public access will be regulated by the university in order to protect privacy and confidentiality concerns, as well to respect any proprietary or intellectual property rights. Administrators will consult with the university's legal office to address any concerns on a case-by-case basis, if necessary. Terms of use will include requirements of attribution along with disclaimers of liability in connection with any use or distribution of the research data, which may be conditioned under some circumstances.

Appendix: Observations of HOT lane capacity

The dataset covers the period from 10/17/2019 to 10/23/2019 from PeMS. We analyzed the data from two freeways: I-680 and I-580 with 3 scenarios. For I-680, we have two scenarios: one isolated HOT lane scenario and two HOT lanes. The road segment with two HOT lanes has seven lanes in total where the road segment with one HOT lane has four lanes in total. For I-580 there is only one HOT lane scenario with 4 lanes. The plots of the fundamental diagram, throughput and speed are shown below. Notably, the holidays and cases with pandemics are not included in our dataset to eliminate disturbances.

I-580 Two HOT lanes

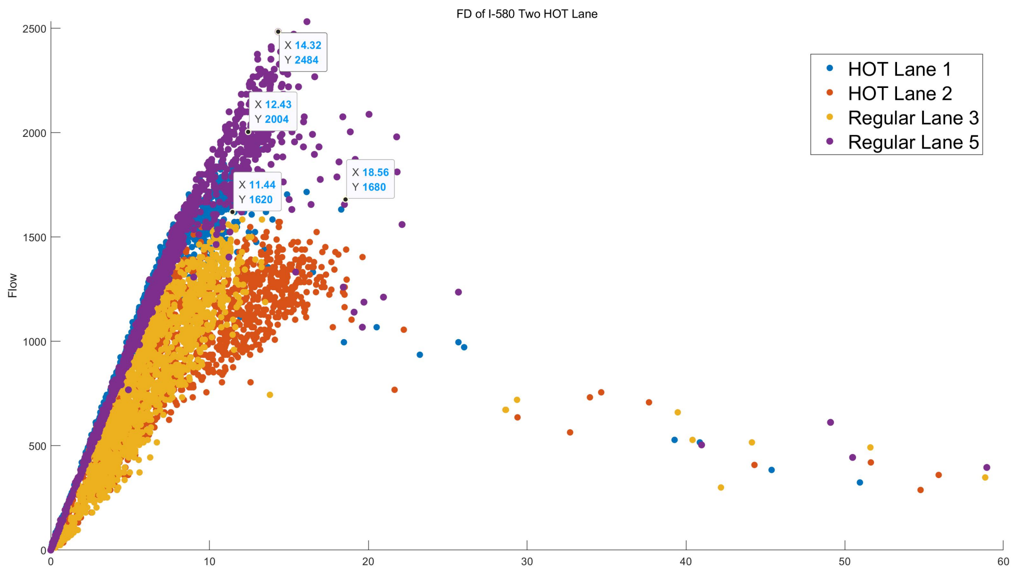

Figure 9: Fundamental Diagram

Lane 1 represents the inside HOT lane and Lane 2 represents the outside HOT lane. In the fundamental diagram, the labels are the maximum throughput for each lane, namely the capacity. The capacity of the outside HOT lane has dropped from nearly 2000 to 1680, where the adjacent regular lane keeps the same capacity with the inside HOT lane (1620 and 1680). Lane 5, where the 5th regular lane has potential large inflow from on-ramp upstream, received a lot of flows and thus the capacity reaches over 2400 vehs/h. The throughput plot shows that the flow of the inside HOT lane always exceeds that of the outside HOT lane. The throughput of the adjacent lane keeps the same flow as the outside HOT lane. But the speed plot shows that the inside HOT lane has a bigger

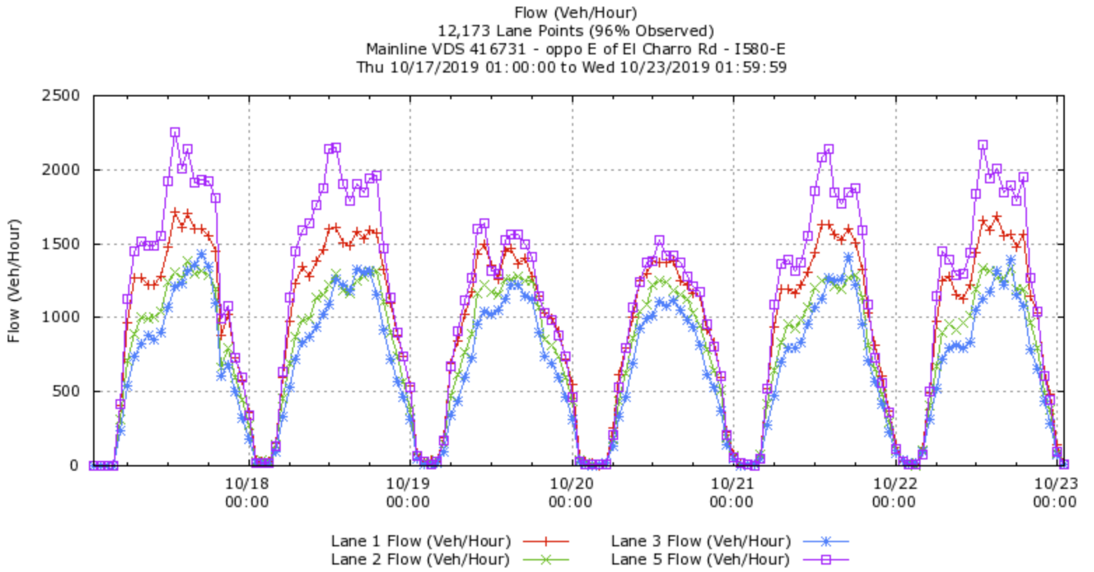

Figure 10: Throughput Over Time

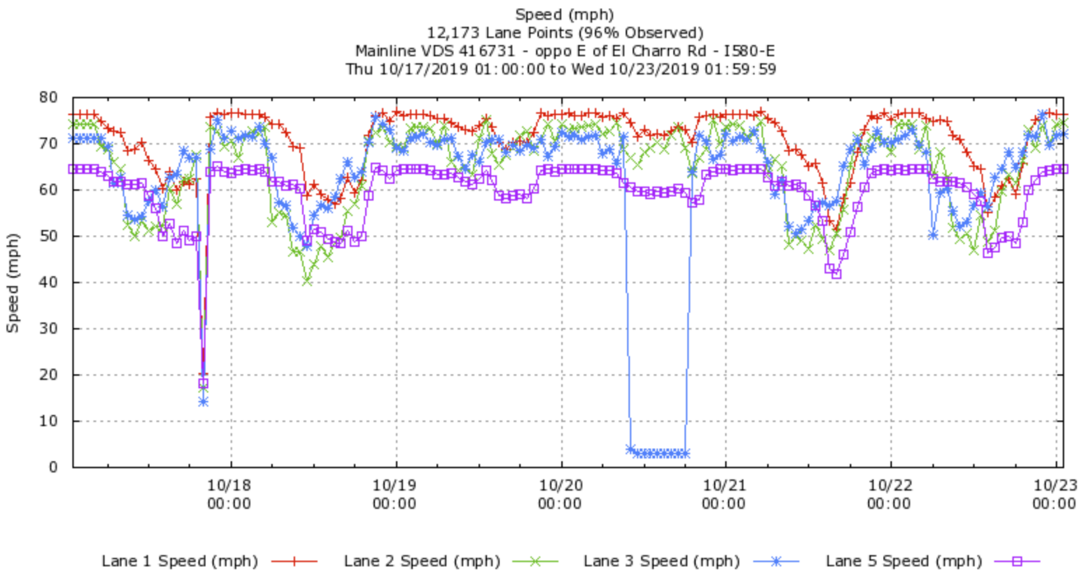

Figure 11: Speed Over Time

average speed compared to the outside HOT lane.

I-580 One HOT lanes

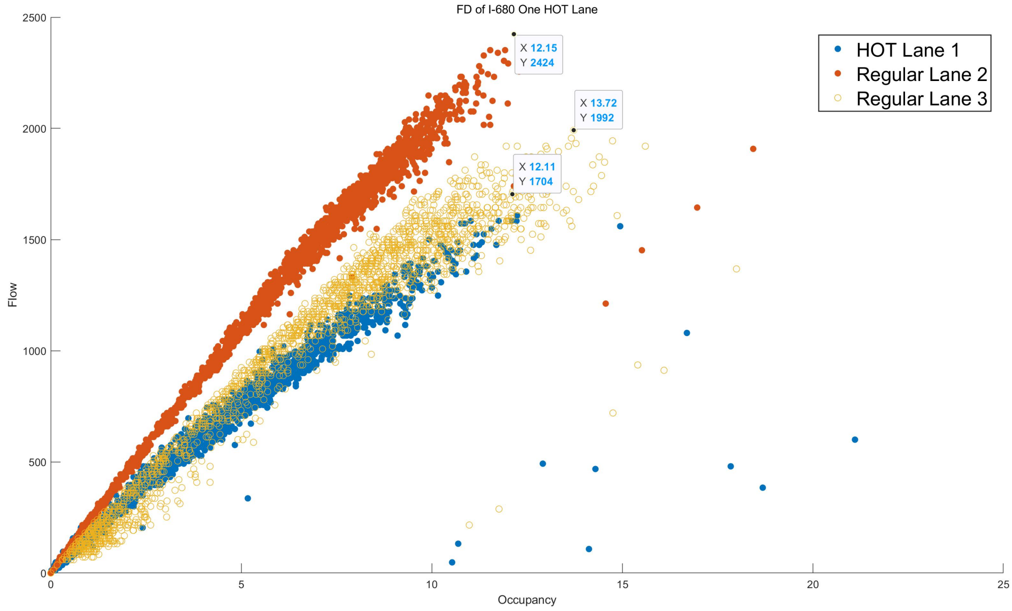

Figure 12: Fundamental Diagram

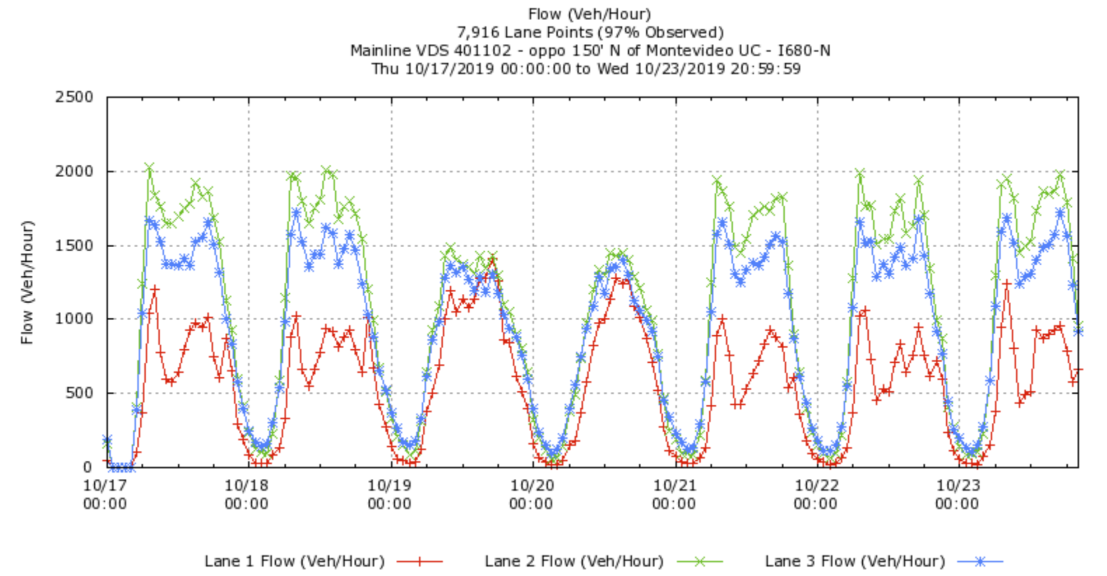

Figure 13: Throughput Over Time

This is the scenario of a single-lane, with a restricted entry HOT lane. The capacity of the single HOT lane is around 1700 vehs/h, where the adjacent GP lane has the same capacity. The throughput of the single-lane,restricted entry HOT lane and the adjacent regular lane are almost

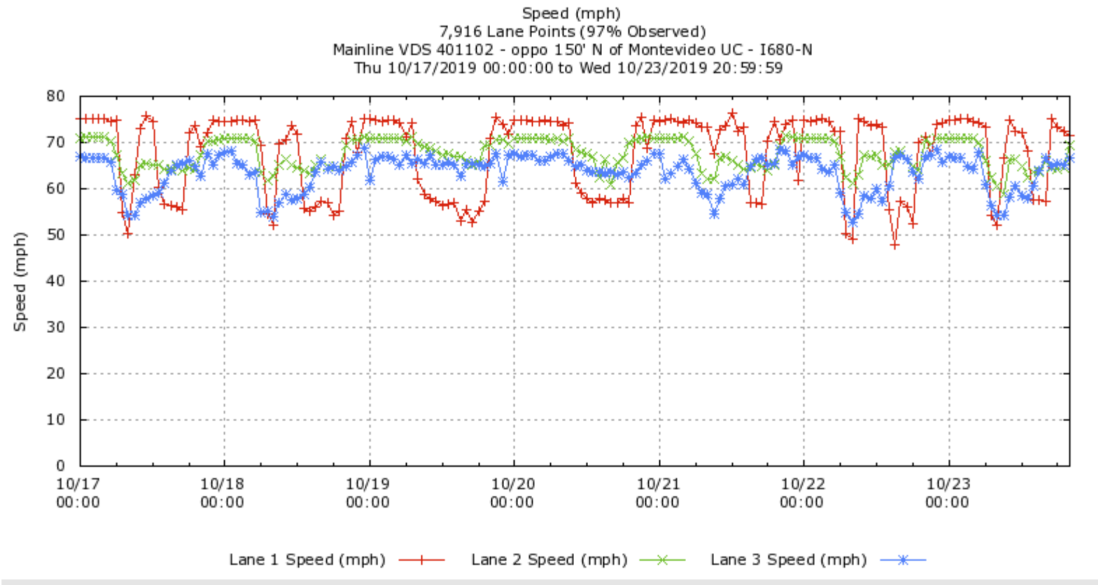

Figure 14: Speed Over Time

the same. The same results can be drawn from other time periods. Also, the single-lane, restricted entry HOT lane always has a higher average speed than the GP lanes, which is consistent with the two HOT lanes' scenario.

I-680 One HOT lanes

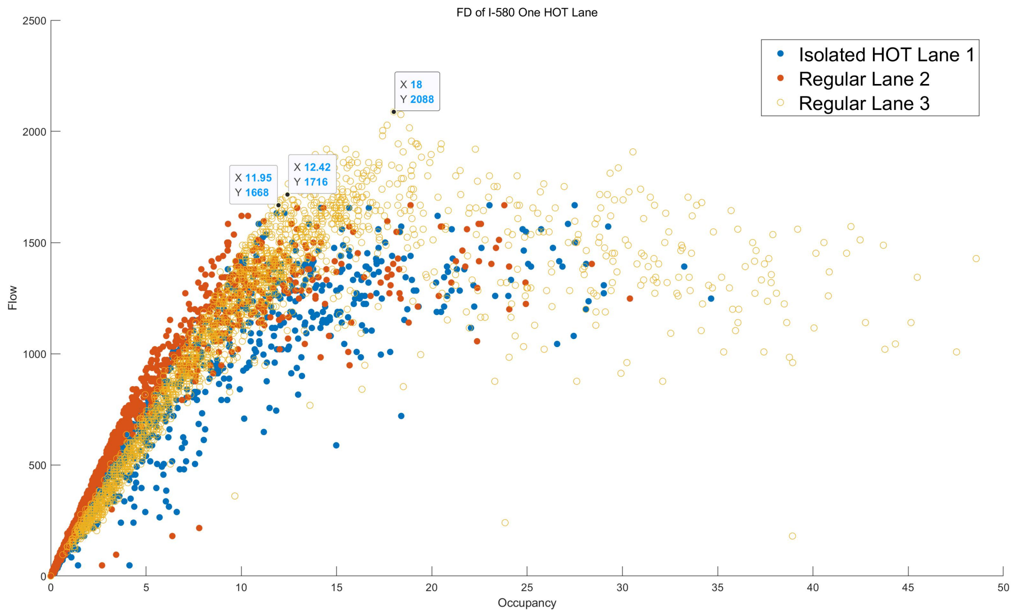

Figure 15: Fundamental Diagram

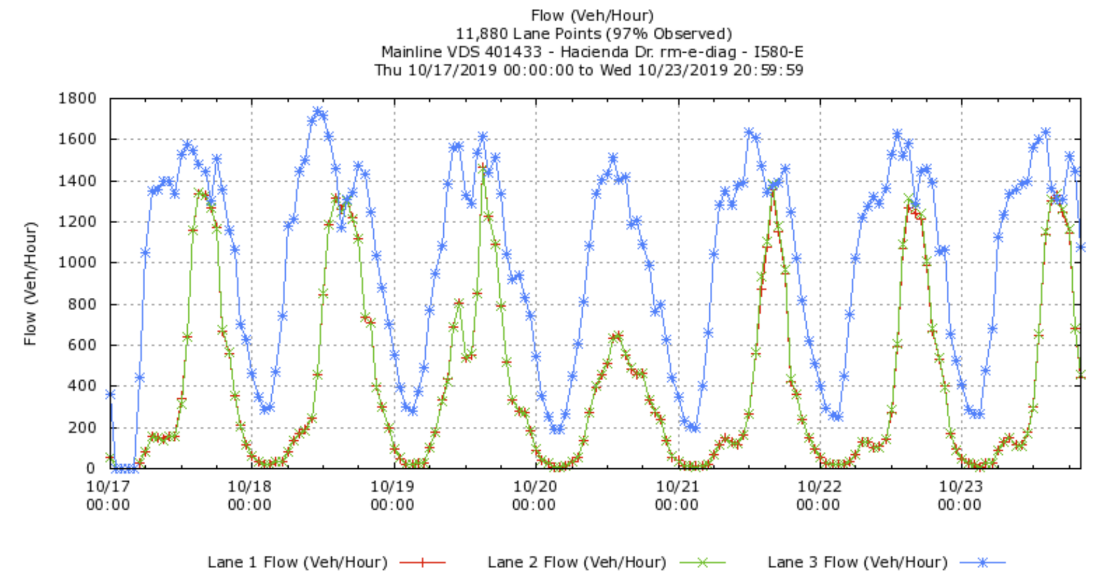

Figure 16: Throughput Over Time

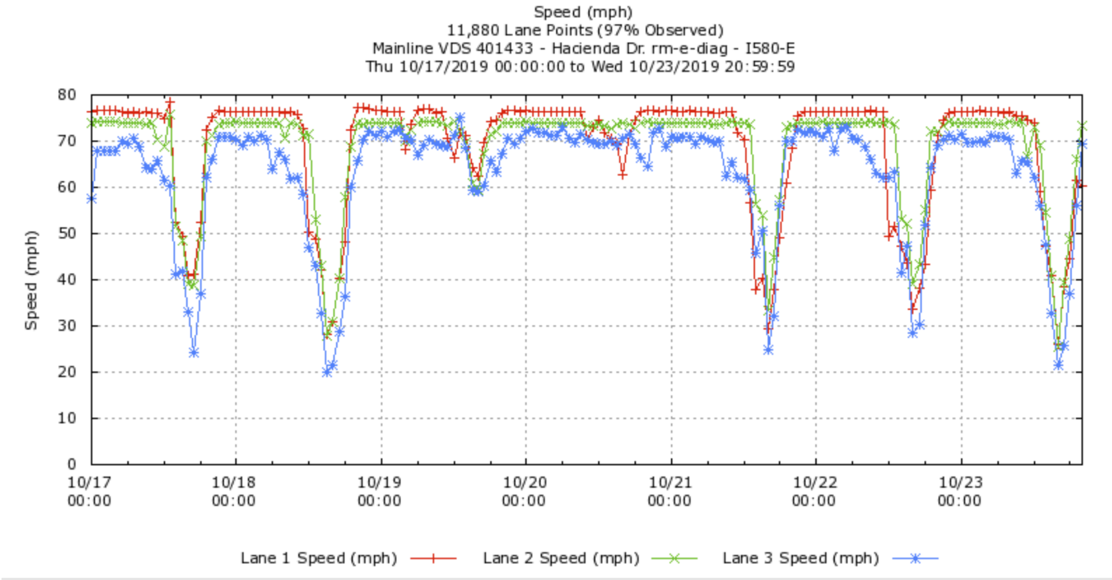

Figure 17: Speed Over Time

When the HOT lane is not isolated. Its capacity reaches 1700 vehs/h. The adjacent regular lane has higher capacity over 2400 vehs/h where the 3rd regular lane drops to 2000. The HOT lane has the highest average speed compared to GP lanes, showing no congestion.

Summary

With two HOT lanes, the capacity of the inside HOT lane is found to be larger than the capacity of the outside HOT lane. However, the capacity of the outside HOT lane, the single-lane, restricted one HOT lane and single-lane, free-entry HOT lane are nearly the same. Based on these empirical findings, we will use a higher per-lane capacity for HOT lanes when a freeway has more than one HOT lane.